

# Exact and Approximate Synthesis of TEM-Mode Transmission-Type Directional Filters

JOHN L. B. WALKER, MEMBER, IEEE

**Abstract**—An exact equivalent circuit is derived for TEM-mode transmission-type directional filters. An approximate synthesis technique is described which provides excellent results and which requires only the use of a hand-held electronic calculator to compute the element values. An exact synthesis technique is also described.

## I. INTRODUCTION

IN SYSTEMS which use frequency division multiplex (FDM), one has to be able to combine and separate the individual channels; directional filters [1] are one type of device capable of performing this function. The feature that distinguishes directional filters from other channel separators, such as diplexers, is that directional filters are perfectly matched at all ports at all frequencies—an important practical advantage.

In this paper an exact equivalent circuit for the TEM-mode transmission-type directional filter will first be determined, and then both an exact and an approximate design method will be described. While the exact method requires a table of element values [2] or a computer, the approximate method requires only an electronic hand-held calculator; the sole disadvantage of the approximate technique is that one can not accurately predict the response of the filter in the very far stopband.

The design of this type of directional filter has also been considered by Hoffman [3]. He considered a generalized form of the network in [3, Fig. 5] in which the directional couplers are allowed to be transversely asymmetric. However, his design technique inherently requires the use of a computer.

## II. ANALYSIS OF A SINGLE-RESONATOR DIRECTIONAL FILTER

Consider the single-resonator directional filter shown in Fig. 1, where the ground plane has been omitted for clarity. This device is basically two transversely symmetrical-directional couplers of characteristic impedance  $Z_0$ , interconnected by two equal-length transmission lines also of characteristic impedance  $Z_0$ . For generality, it is assumed that the two directional couplers are not identical, i.e.,

$$Z_{0e}^a \neq Z_{0e}^b, \quad Z_{0o}^a \neq Z_{0o}^b$$

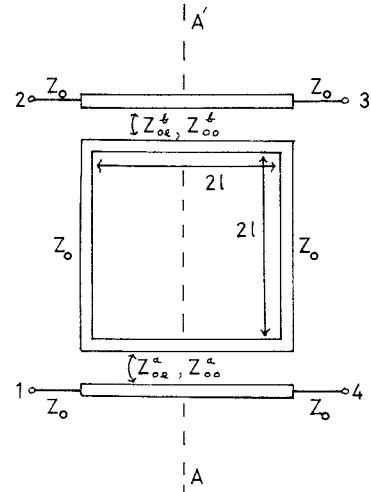


Fig. 1. A single-resonator directional filter.

since this flexibility is needed in the realization of the even-ordered equi-ripple response. If the network is considered as a redrawn  $n$ -wire line network [4], then pure TEM-mode propagation can be assumed.

The scattering matrix of this network is given by [5]

$$\begin{aligned} S_{11} &= S_{44} = \frac{1}{2}(\Gamma_e^a + \Gamma_o^a) \\ S_{22} &= S_{33} = \frac{1}{2}(\Gamma_e^b + \Gamma_o^b) \\ S_{12} &= S_{34} = \frac{1}{2}(\tau_e + \tau_o) \\ S_{13} &= S_{24} = \frac{1}{2}(\tau_e - \tau_o) \\ S_{14} &= \frac{1}{2}(\Gamma_e^a - \Gamma_o^a) \\ S_{23} &= \frac{1}{2}(\Gamma_e^b - \Gamma_o^b) \end{aligned} \quad (1)$$

where  $\Gamma_{e,o}^{a,b}$  and  $\tau_{e,o}$  are the reflection and transmission scattering parameters of the even- and odd-mode networks with respect to the plane of symmetry  $AA'$ .

The even-mode network equivalent circuit [6] is shown in Fig. 2(a), where

$$t' = \tanh \frac{l}{v} p \quad (2)$$

$v$  is the velocity of TEM wave propagation and  $p = \sigma + j\omega$ , the complex frequency variable.

Let

$$t = \tanh \frac{l}{v} p \quad (3)$$

then

$$t = \frac{2t'}{1 + t'^2}. \quad (4)$$

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The author is with the General Electric Co., Ltd., Hirst Research Centre, East Lane, Wembley, England.

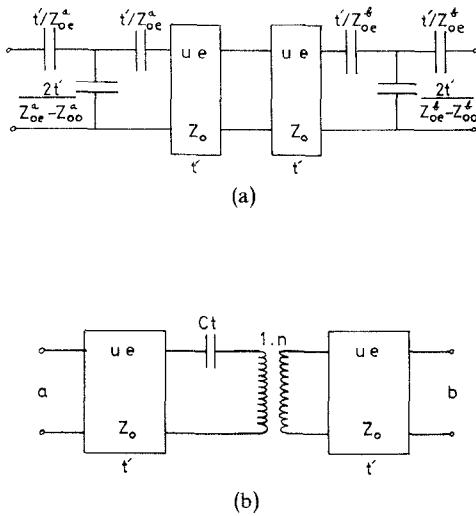


Fig. 2. Single-resonator even-mode equivalent circuits.

Application of Kuroda's transformations [6] followed by use of the identity (4) above shows that the network in Fig. 2(b) is also an exact equivalent circuit for the even-mode network of Fig. 1 if

$$n = \left( \frac{Z_{0e}^b + Z_{0o}^b + 2Z_0}{Z_{0e}^a + Z_{0o}^a + 2Z_0} \right) \cdot \left( \frac{Z_{0e}^a - Z_{0o}^a}{Z_{0e}^b - Z_{0o}^b} \right) \quad (5)$$

$$\frac{1}{C} = 4Z_0^2 \left( \frac{Z_{0e}^a + Z_{0o}^a + 2Z_0}{Z_{0e}^a - Z_{0o}^a} \right)^2 \cdot \left( \frac{Z_{0o}^a}{(Z_0 + Z_{0o}^a)^2} + \frac{Z_{0e}^b}{(Z_0 + Z_{0o}^b)^2} \right). \quad (6)$$

In the usual case of two identical couplers, these equations simplify considerably to

$$n = 1, \quad C = \frac{Z_{0e} + Z_{0o} - 2Z_0}{8Z_0^2}. \quad (7)$$

For this case the directional filter is doubly symmetrical.

Equations (5) and (6) are the analysis equations, the corresponding synthesis ones being

$$Z_{0e}^a Z_{0o}^a = Z_{0e}^b Z_{0o}^b = Z_0^2 \quad (8)$$

$$k_a^2 = \frac{8n^2 CZ_0(n^2 + CZ_0 + n^2 CZ_0)}{(n^2 + CZ_0 + 3n^2 CZ_0)^2} \quad (9)$$

$$k_b^2 = \frac{8CZ_0(n^2 + CZ_0 + n^2 CZ_0)}{(n^2 + 3CZ_0 + n^2 CZ_0)^2} \quad (9)$$

where  $k_i^2$  is the power coupling coefficient of the directional coupler given by

$$k_i^2 = \left( \frac{Z_{0e}^i - Z_{0o}^i}{Z_{0e}^i + Z_{0o}^i} \right)^2, \quad i = a \text{ or } b. \quad (10)$$

The odd-mode network equivalent circuit is identical to that of the even-mode network except for the fact that the series capacitor in Fig. 2(b) is replaced by a shunt inductor of value

$$L = CZ_0^2 \quad (11)$$

and the transformer's turns ratio is now  $1:(1/n)$ .

Hence  $\Gamma_e^{a,b} = -\Gamma_o^{a,b}$  and  $\tau_e = \tau_o$ . Thus from (1)

$$\begin{aligned} S_{11} &= 0 \\ S_{12} &= \tau_e \\ S_{13} &= 0 = S_{24} \\ S_{14} &= \Gamma_e^a \end{aligned} \quad (12)$$

i.e., the device is perfectly matched at all ports at all frequencies and provides isolation between ports 1 and 3 and between ports 2 and 4.

Since the device is lossless,

$$|\Gamma_e^a|^2 + |\tau_e|^2 = 1.$$

Therefore, if a signal is incident at port 1, part of that signal will emerge at port 2 and the remainder at port 4. Furthermore, this energy division is frequency selective, and all the incident signal at port 1 emerges from port 2 and none at port 4 when  $n = 1$  and the resonator is one wavelength long or any multiple thereof. Thus in FDM systems the dropped channel appears at port 2 if the complete FDM signal is applied to port 1 since  $|S_{12}|^2$  has a bandpass amplitude characteristic, while the remaining signal emerges from port 4.

If the maximum practically achievable coupling in the directional couplers is taken as 3 dB for an air-filled coaxial system, then  $CZ_0$  in (7) lies in the range

$$0 \leq CZ_0 \leq 0.1. \quad (13)$$

Straightforward calculation shows that when  $n = 1$

$$|S_{12}|^2 = |\tau_e|^2 = \frac{4C^2 Z_0^2 \tan^2 \theta}{1 + 4C^2 Z_0^2 \tan^2 \theta} \quad (14)$$

where  $\theta = 2l\omega/v$ . Hence the maximum 3-dB bandwidth of the dropped channel is  $90^\circ \pm 11^\circ$ , i.e., 24 percent.

### III. ANALYSIS OF A MULTIRESONATOR DIRECTIONAL FILTER

If  $n$  single-resonator directional filters are cascaded as shown in Fig. 3, then the even-mode equivalent circuit is that shown in Fig. 4. The connections between the directional filters must be of zero electrical length, otherwise there will be additional directional filters in the cascade other than those intended. It can be proved that the cascade of perfectly matched and isolated networks in Fig. 3 is itself perfectly matched and isolated; physically, this fact is obvious since there is no mismatch at any of the internal ports in Fig. 3. Hence (12) apply for this network also, the dropped channel response being determined by the transmission scattering parameter  $\tau_e$  of the even-mode network in Fig. 4.

It is proved in the Appendix that a cascade of two-directional couplers of the form illustrated in Fig. 3, each with characteristic impedance  $Z_0$ , is equivalent to a single-directional coupler also of characteristic impedance  $Z_0$ . If the individual couplers have power coupling coefficients  $k_1^2$

and  $k_2^2$ , then the equivalent-directional coupler has a power coupling coefficient of

$$k^2 = \left( \frac{k_1 k_2}{1 + \sqrt{1 - k_1^2} \sqrt{1 - k_2^2}} \right)^2 \quad (15)$$

where

$$k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}. \quad (16)$$

Furthermore, it can be shown that

$$k \leq k_1 \quad \text{and} \quad k \leq k_2 \quad (17)$$

so the line separation for the equivalent directional coupler is greater than the line separation of either of the two constituent directional couplers. Hence the network in Fig. 5, which is the conventional TEM-mode transmission-type directional filter [1], is an exact equivalent of the network in Fig. 3.

#### IV. EXACT SYNTHESIS

The first and last half-length unit elements (u.e.) in Fig. 4 can be completely ignored in the design process since they have no effect upon the amplitude response and merely add a constant to the group delay response. Hence an  $n$ -resonator directional filter has  $n$  capacitive stubs and  $(n - 1)$  unit elements of characteristic impedance  $Z_0$ , all in the frequency variable  $t$ . However, application of Kuroda's transformations shows that only one of the stubs is electrically effective. Hence the appropriate approximating functions [7] for a multiresonator transmission-type directional filter are either

$$|S_{12}|^2 = |\tau_e|^2 = \frac{1}{1 + \left( \frac{\tan \theta_c}{\tan \theta} \right)^2 \left( \frac{\cos \theta}{\cos \theta_c} \right)^{2n}} \quad (18)$$

for a maximally flat dropped-channel response, or

$$|S_{12}|^2 = |\tau_e|^2 = \frac{1}{1 + \varepsilon^2 \left[ T_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) T_n \left( \frac{\cos \theta}{\cos \theta_c} \right) - U_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) U_n \left( \frac{\cos \theta}{\cos \theta_c} \right) \right]^2} \quad (19)$$

for an equi-ripple dropped-channel response.  $T_n$  and  $U_n$  are the Chebyshev polynomials of the first and second kinds, respectively. Although no explicit element value formulae are known for these responses, tables of element values [2] for selected ripple levels have been produced.

The design procedure can be summarized as follows.

- 1) Determine  $\theta_c$  and  $n$  in (18) or (19) from the bandwidth and stopband attenuation specifications.
- 2) Determine the element values from the appropriate set of tables in [2].
- 3) Apply a capacitance matrix transformation [8] and then impedance scale so that every u.e., apart from the central one in the even-ordered equi-ripple case, has a characteristic impedance of  $Z_0$ .
- 4) Determine  $k_i^2$  from (9). With the exception of the even-ordered equi-ripple case, no transformers are needed in Fig. 4, so every directional filter in the cascade shown in Fig. 3 is doubly symmetric and  $k_a = k_b$ . For the even-ordered

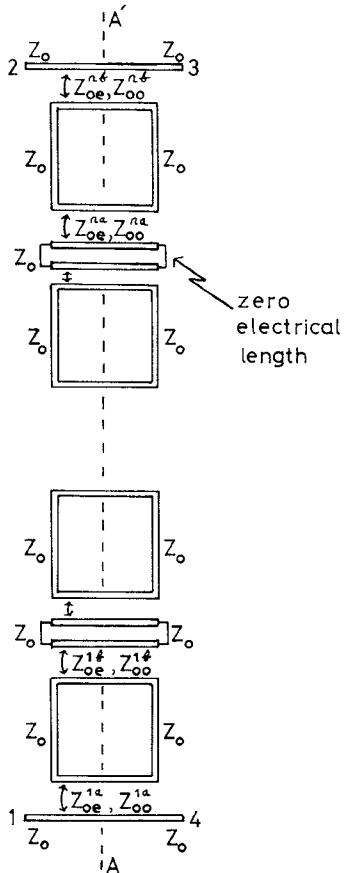


Fig. 3. Multiresonator transmission-type directional filter.

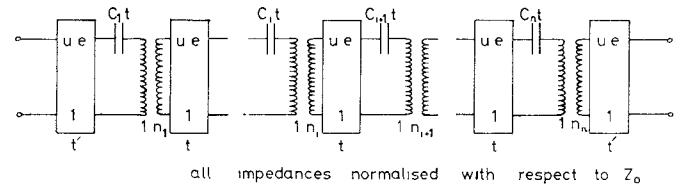


Fig. 4. Even-mode equivalent circuit of multiresonator directional filter.

$$\frac{1}{1 + \varepsilon^2 \left[ T_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) T_n \left( \frac{\cos \theta}{\cos \theta_c} \right) - U_1 \left( \frac{\tan \theta_c}{\tan \theta} \right) U_n \left( \frac{\cos \theta}{\cos \theta_c} \right) \right]^2} \quad (19)$$

equi-ripple case, only the two central directional couplers in the cascade in Fig. 3 are not doubly symmetric and require transformers.

5) Determine the power coupling coefficient of the equivalent single-directional coupler from (15), and then determine the  $Z_{0e}^{i,i+1}$  of Fig. 5 from (16).

6) Finally, determine the physical dimensions of the device. If a realization in the form of rectangular bars between parallel ground planes is required, then the separation between the resonators and the widths of the bars in Fig. 5 can be determined by using the results of Getsinger [9] once the  $Z_{0e}^{i,i+1}$  are known; the center frequency determines the length  $2l$ .

#### V. APPROXIMATE SYNTHESIS

An exact equivalent circuit [10] for a u.e. is shown in Fig. 6 where all impedances have been normalized to the charac-

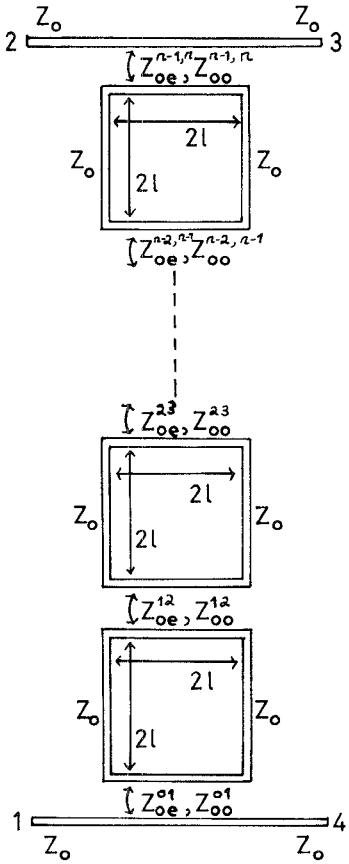


Fig. 5. Standard multiresonator transmission-type directional filter.

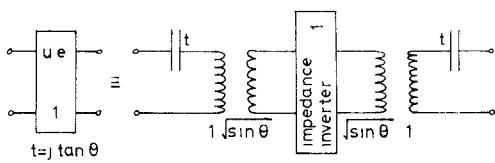


Fig. 6. Exact equivalent circuit for a unit element.

teristic impedance of the u.e. If this equivalence is used in Fig. 4 then, after simplification, the network of Fig. 7 is an exact equivalent circuit for the even-mode network of Fig. 4 with

$$\begin{aligned} Y_1 &= \frac{jC_1 n_1^2}{(n_1^2 + C_1) \cos \theta} \\ Y_i &= \frac{jC_i n_i^2}{(n_i^2 + C_i + n_i^2 C_i) \cos \theta} \\ Y_n &= \frac{jC_n}{(1 + C_n) \cos \theta}, \quad i = 2 \rightarrow n - 1. \end{aligned} \quad (20)$$

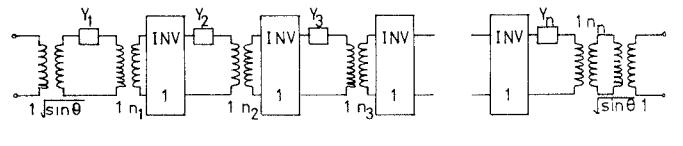
The first and last half-length unit elements have been neglected as before. Since the value of  $\sqrt{\sin \theta}$  is in the range

$$0.99 < \sqrt{\sin \theta} \leq 1.0 \quad (21)$$

for

$$79^\circ < \theta < 101^\circ \quad (22)$$

the transformers in Fig. 7 have a negligible effect upon the passband amplitude response of the network.



all impedances normalised with respect to  $Z_o$

Fig. 7. Alternative exact even-mode equivalent circuit for a multi-resonator directional filter.

Now consider the basic distributed lowpass prototype network with cut-off frequency  $\theta = 45^\circ$  shown in Fig. 8 where  $t = j \tan \theta$ . For a maximally flat response

$$|S_{12}|^2 = \frac{1}{1 + \tan^{2n} \theta} \quad (23)$$

and the element values are given by [11]

$$L_i = 2 \sin \left[ \frac{2i - 1}{2n} \pi \right], \quad i = 1 \rightarrow n \quad (24)$$

$$K_{i,i+1} = 1, \quad i = 1 \rightarrow n - 1. \quad (25)$$

For an equi-ripple response

$$|S_{12}|^2 = \frac{1}{1 + \varepsilon^2 T_n^2 (\tan \theta)} \quad (26)$$

and the element values are given by [11]

$$L_1 = \frac{2 \sin (\pi/2n)}{\sinh \left[ \frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon} \right]}$$

$$L_i L_{i+1} = \frac{4 \sin [(2i - 1)\pi/2n] \sin [(2i + 1)\pi/2n]}{\sinh^2 \left[ \frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon} \right] + \sin^2 [i\pi/n]}, \quad i = 1 \rightarrow n - 1$$

$$K_{i,i+1} = 1, \quad i = 1 \rightarrow n - 1, n \text{ odd}$$

$$K_{i,i+1} = 1, \quad i \neq n/2, n \text{ even}$$

$$K_{i,i+1} = (-1)^{n/2+1} \varepsilon + \sqrt{1 + \varepsilon^2}, \quad i = n/2, n \text{ even.} \quad (27)$$

Examination of Figs. 7 and 8 shows that to convert the lowpass prototype into the even-mode equivalent circuit (ignoring the frequency-dependent transformers) one must apply the frequency transformation

$$j \tan \theta \rightarrow -j \frac{\cos \theta}{\cos \theta_c} \quad (28)$$

where  $\cos \theta_c$  is a bandwidth scaling factor. Hence the directional filter's dropped-channel response is given approximately by

$$|S_{12}|^2 = \frac{1}{1 + \left( \frac{\cos \theta}{\cos \theta_c} \right)^{2n}} \quad (29)$$

for a maximally flat response, and by

$$|S_{12}|^2 = \frac{1}{1 + \varepsilon^2 T_n^2 \left( \frac{\cos \theta}{\cos \theta_c} \right)} \quad (30)$$

for an equi-ripple response. Equations (29) and (30) cannot be used to predict the frequency response of the even-mode

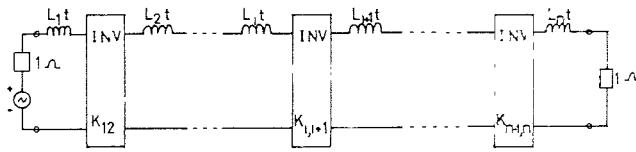


Fig. 8. Basic distributed lowpass prototype network.

network accurately in the very far stopband since they all predict a finite insertion loss at  $\omega = 0$ .

Applying the frequency transformation (28) the elements in Fig. 7 are related to those in Fig. 8 by

$$\begin{aligned} Y_1 &= \frac{j \cos \theta_c}{L_1 \cos \theta}, & n_1 &= \frac{1}{\sqrt{K_{12}}} \\ Y_i &= \frac{j K_{i-1,i} \cos \theta_c}{L_i \cos \theta}, & n_i &= \sqrt{\frac{K_{i-1,i}}{K_{i,i+1}}} \\ Y_n &= \frac{j K_{n-1,n} \cos \theta_c}{L_n \cos \theta}, & n_n &= \sqrt{K_{n-1,n}}, \\ & & i &= 2 \rightarrow n-1. \end{aligned} \quad (31)$$

Combining these equations with (20), and using the values of  $K_{i,i+1}$  given in (25) and (27), one has the following design formulae for the elements in Fig. 4 for the maximally flat and odd-ordered equi-ripple response:

$$\begin{aligned} C_i &= \frac{\cos \theta_c}{L_i - \cos \theta_c}, & i &= 1 \text{ or } n \\ C_i &= \frac{\cos \theta_c}{L_i - 2 \cos \theta_c}, & i &= 2 \rightarrow n-1 \\ n_i &= 1, & i &= 1 \rightarrow n. \end{aligned} \quad (32)$$

For the even-ordered equi-ripple response the design formulae are

$$\begin{aligned} C_1 &= \frac{C_2}{K_{12}} = \frac{\cos \theta_c}{L_1 - K_{12} \cos \theta_c} \\ n_1 &= 1/n_2 = 1/\sqrt{K_{12}}, & n &= 2. \end{aligned} \quad (33)$$

If  $n \neq 2$ , then

$$\begin{aligned} C_i &= \frac{\cos \theta_c}{L_i - \cos \theta_c}, & i &= 1, n \\ C_i &= \frac{\cos \theta_c}{L_i - 2 \cos \theta_c}, & i &= 2 \rightarrow \frac{n}{2} - 1, \quad \frac{n}{2} + 2 \rightarrow n-1 \\ C_{n/2} &= \frac{C_{n/2+1}}{K_{n/2,n/2+1}} = \frac{\cos \theta_c}{L_{n/2} - (1 + K_{n/2,n/2+1}) \cos \theta_c}, \\ n_i &= 1, \quad i \neq \frac{n}{2}, \frac{n}{2} + 1 \\ n_{n/2} &= \frac{1}{n_{n/2+1}} = 1/K_{n/2,n/2+1}. \end{aligned} \quad (34)$$

Having determined the  $C_i$  and  $n_i$  of Fig. 4, the last three steps of the exact synthesis procedure must be performed.

It should be noted that for the maximally flat response the minimum value of  $L_i$  occurs when  $i = 1$  or  $n$ . This conclusion is approximately true for the equi-ripple response as well unless the ripple level is large. Thus from (32), (33), or (34) the maximum value of  $C_i$  occurs when  $i = 1$  or  $n$ , but it was shown in Section II that  $C_i$  is bounded by  $0 \leq C_i \leq 0.1$  for practical realizability. Hence we require

$$\frac{\cos \theta_c}{L_1} < \frac{1}{11}. \quad (35)$$

Once  $n$  and  $\theta_c$  have been determined for any design, one should first check that this inequality is satisfied in order to ensure that it will be physically possible to construct the directional filter.

A more obvious approximate design procedure is to approximate the unit elements in the even-mode equivalent circuit of Fig. 4 by impedance inverters, and then equate this network with the basic distributed lowpass prototype shown in Fig. 8 after applying the frequency transformation

$$t \rightarrow 1/t.$$

However, this method results in considerable distortion of the equi-ripple response.

## VI. EXAMPLE

Determine the values of the elements in Fig. 4 for a directional filter with a dropped-channel response covering 5.64–6.36 GHz with 0.1-dB ripple and attenuation  $> 30$  dB at 5.33 GHz.

The center frequency is 6 GHz, corresponding to  $\theta = 90^\circ$ ,  $\theta_c = 84.6^\circ$ , and  $\epsilon^2 = 0.023$ . From (30),  $n = 5$  is sufficient to provide over 30-dB attenuation at 5.33 GHz. From (27) we have

$$\begin{aligned} L_1 &= L_5 = 1.147 \\ L_2 &= L_4 = 1.371 \\ L_3 &= 1.975 \\ K_{12} &= K_{23} = K_{34} = K_{45} = 1. \end{aligned}$$

Using these values in (32) yields the following element values (the element values in brackets are the values determined from the exact synthesis technique of Section IV):

$$\begin{aligned} C_1 &= C_5 = 0.0894 \quad (0.0894) \\ C_2 &= C_4 = 0.0796 \quad (0.0793) \\ C_3 &= 0.0527 \quad (0.0526) \\ n_1 &= n_2 = n_3 = n_4 = n_5 = 1 \quad (1). \end{aligned}$$

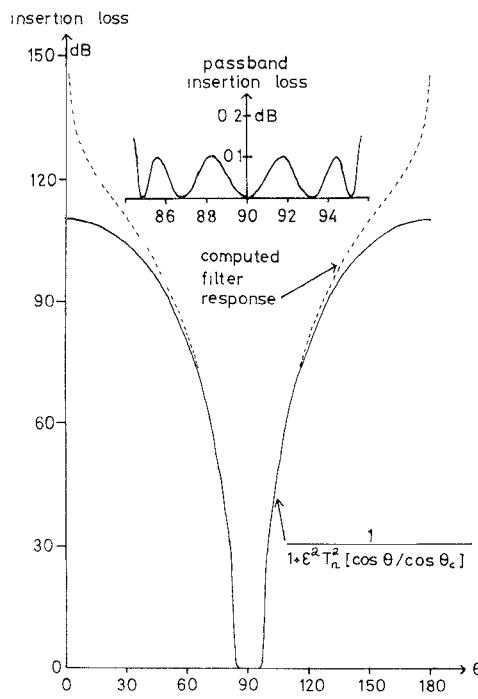


Fig. 9. Computed dropped-channel response of a directional filter

Hence, the approximate synthesis technique yields element values with a maximum error of 0.4 percent in this case.

Since the inequality (35) is satisfied, this specification is realizable in practice.

The computed dropped-channel response of this filter is shown in Fig. 9 together with the response predicted by (30). There is no discernible departure from an equi-ripple response in the passband but, because of the presence of the frequency-dependent transformers, the filter has more attenuation than predicted in the far stopband.

If the bandwidth is increased from 12 percent to 20 percent with the same values of  $n$  and  $\epsilon$ , then the maximum error in the element values calculated by the approximate technique is only 0.5 percent. However, this directional filter is not practically realizable since the inequality (35) is no longer satisfied. Hence the limitation is not that the approximate technique becomes too inaccurate as the bandwidth increases, but that the element values become unrealizable.

## VII. CONCLUSIONS

An exact equivalent circuit has been derived for the multi-resonator directional filter. An exact design technique has been outlined which requires either a table of element values or a computer. An approximate design technique has also been described which produces excellent results and which requires only the use of a hand-held electronic calculator to compute the element values.

## APPENDIX

Consider the cascade of two-directional couplers shown in Fig. 10 where it is assumed that they both have the same characteristic impedance. These have scattering matrices given by [5]

$$(S_i) = \begin{bmatrix} 0 & \Gamma_i & 0 & \tau_i \\ \Gamma_i & 0 & \tau_i & 0 \\ 0 & \tau_i & 0 & \Gamma_i \\ \tau_i & 0 & \Gamma_i & 0 \end{bmatrix}, \quad i = 1 \text{ or } 2 \quad (36)$$

where

$$\begin{aligned} \Gamma_i &= \frac{j k_i \sin \theta}{\sqrt{1 - k_i^2} \cos \theta + j \sin \theta} \\ \tau_i &= \frac{\sqrt{1 - k_i^2}}{\sqrt{1 - k_i^2} \cos \theta + j \sin \theta}. \end{aligned} \quad (37)$$

Define a matrix  $(R_i)$  by

$$\begin{bmatrix} b_1^i \\ b_4^i \\ a_1^i \\ a_4^i \end{bmatrix} = (R_i) \begin{bmatrix} a_2^i \\ a_3^i \\ b_2^i \\ b_3^i \end{bmatrix} \quad (38)$$

where  $a_j^i$  is the incident wave at port  $j$  and  $b_j^i$  is the reflected wave at port  $j$ . Then

$$(R_i) = \begin{bmatrix} \alpha_i & 0 & 0 & \beta_i \\ 0 & \alpha_i & \beta_i & 0 \\ 0 & -\beta_i & \gamma_i & 0 \\ -\beta_i & 0 & 0 & \gamma_i \end{bmatrix}$$

where

$$\begin{aligned} \alpha_i &= \frac{\Gamma_i^2 - \tau_i^2}{\Gamma_i} \\ \beta_i &= \tau_i/\Gamma_i, \quad \gamma_i = 1/\Gamma_i. \end{aligned} \quad (39)$$

Now the  $[R]$  matrix for the cascade of the two-directional couplers is the product of the individual  $[R]$  matrices

$$[R] = [R_1][R_2]. \quad (40)$$

Straightforward matrix multiplication shows that  $[R]$  is of the same form as  $[R_i]$ , and that

$$\gamma = \frac{1}{\Gamma} = \frac{1 - \tau_1 \tau_2}{\Gamma_1 \Gamma_2}. \quad (41)$$

Thus

$$\Gamma = \frac{-k_1 k_2 \sin^2 \theta}{(\sqrt{1 - k_1^2} \cos \theta + j \sin \theta)(\sqrt{1 - k_2^2} \cos \theta + j \sin \theta) - \sqrt{1 - k_1^2} \sqrt{1 - k_2^2}}. \quad (42)$$

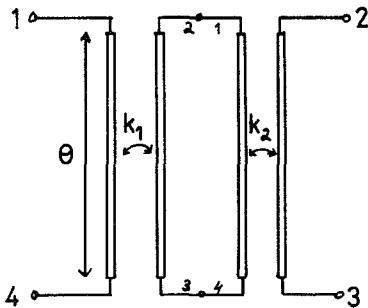


Fig. 10. A cascade of two-directional couplers.

Simplifying,

$$\Gamma = \frac{jk \sin \theta}{\sqrt{1 - k^2} \cos \theta + j \sin \theta} \quad (43)$$

where

$$k = \frac{k_1 k_2}{1 + \sqrt{1 - k_1^2} \sqrt{1 - k_2^2}}. \quad (44)$$

Hence the cascade of two-directional couplers in Fig. 10 is equivalent to a single-directional coupler with coupling coefficient given by (44).

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# A New MIC Doppler Module

SHUTARO NANBU

**Abstract**—A new microwave integrated circuit (MIC) Doppler module comprising a germanium avalanche oscillator diode and a Schottky-barrier detector diode has been fabricated and analyzed. The module is essentially a high-stability MIC oscillator, which is connected to an X-band waveguide by means of a stripline probe. In addition to high minimum detectable signal level -100 dBm, a prominent feature that has been revealed is that the Doppler signal level is practically independent of oscillator output power under a specific condition. It is also shown that a simple adaptor attached to the module can provide a direction-sensitive device in a compact form.

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The author is with the Research Laboratory of Matsushita Electronics Corporation, Takatsuki, Osaka, Japan.

#### I. INTRODUCTION

THE ADVENT of low-power-consumption negative resistance diodes useful at microwave frequencies has provided an impetus to developing low-cost light-weight Doppler modules [1], [2]. The low-cost Doppler modules are found in three types: 1) self-mixing Doppler sensors which employ Gunn and IMPATT oscillators [3], 2) a simple-structure Doppler module [4] which consists simply of an oscillator and a detector, and 3) a complex-structure Doppler module which consists of an oscillator, a detector, and a circulator or a directional coupler [1]. As for the first type, the merit of the simplest construction is counterbalanced by the demerit of the lowest sensitivity operation with the